ANALYSIS OF VERTEX BIMAGIC TOTAL LABELINGS IN RELATION TO COMPLEMENTARY PERFECT DOUBLE CONNECTED DOMINATION NUMBER

S. Arunkumar*, R. Senthil Amutha

* Research Scholar, Depatment of Mathematics, Sree Saraswathi Thyagaraja College, Pollachi, Tamilnadu, India
Head & Assistant Professor, Depatment of Mathematics, Sree Saraswathi Thyagaraja College, Pollachi, Tamilnadu, India

ABSTRACT

A set S is a complementary perfect triple connected dominating set, if S is a triple connected dominating set of G and the induced sub graph $< V - S >$ has a perfect matching. The complementary perfect triple connected domination number $\gamma_{cptc}(G)$ is the minimum cardinality taken over all complementary perfect triple connected dominating sets in G. A graph labeling is a one to one function that carries a set of elements onto set of integers called labels. Here we discuss some standard graphs which has vertex bimagic labelings and for which complementary perfect triple connected domination number exists.

KEYWORDS: Dominating set, CPTC Dominating set, VBTL and Standard graphs.

INTRODUCTION

By a graph we mean a finite, simple, connected and undirected graph $G(V, E)$,where V denotes its vertex set and E its edge set. Unless otherwise stated, the graph G has p vertices and q edges. Degree of a vertex v is denoted by (v) , the maximum degree of a graph G is denoted by (G) . We denote a cycle on p vertices by C_p, a path on p vertices by P_p, and a complete graph on p vertices by K_P . A graph G is connected if any two vertices of G are connected by a path. A maximal connected subgraph of a graph G is called a component of G. The number of components of G is denoted by (G) . The complement of G is the graph with vertex set V in which two vertices are adjacent if and only if they are not adjacent in G. A tree is a connected acyclic graph. A bipartite graph (or bigraph) is a graph whose vertices can be divided into two disjoint sets U and V such that every edge connects a vertex in U to one in . The chromatic number of a graph G, denoted by $\chi(G)$ is the smallest number of colors needed to colour all the vertices of a graph G in which adjacent vertices receive different colour[1].A subset S of V is called a dominating set of G if every vertex in $V - S$ is adjacent to at least one vertex in S. The domination number $\gamma(G)$ of G is the minimum cardinality taken over all dominating sets in . A dominating set S of a connected graph G is said to be a connected dominating set of G if the induced sub graph $< S >$ is connected. The minimum cardinality taken over all connected dominating set is the connected domination number and is denoted by γ_c. A set $S \subseteq V$ is a complementary perfect triple connected dominating set if S is a triple connected dominating set of G and the induced subgraph $V - S$ has a perfect matching. The complementary perfect triple connected (CPTC) domination number $\gamma_{cptc}(G)$ is the minimum cardinality taken over all complementary perfect triple connected dominating sets in $G[4][5][6]$.

DEFINITIONS

Cycle graph

A graph consisting of a cycle with n points and by a path $P_n$with n points is called Cycle graph, it is denoted by C_n.

Complete bipartite graph

A Complete bipartite graph is a special kind of bipartite graph where every vertex of the first set is connected to every vertex of the second set. The complete bipartite graph with partitions of order $|V_1| = m$ and $|V_2| = n$ is denoted $V_{m,n}$.

Friendship graph

The Friendship graph F_n can be constructed by joining n copies of the cycle graph C_3 with a common vertex.

Wagner graph

A graph is a 3-regular graph with 8 vertices and 12 edges is called the Wagner graph.

Book graph
The Book graph $S_2 \Theta S_n$ is a connected graph obtained by adding 'n' number of C_4 with one edge. It has $2n$ vertices and $3n - 2$ edges[3].

Vertex bimagic total labelling(VBTL)
Labeling of graphs subject to certain conditions gave raise to enormous work which is listed by J. A. Gallian. In general, by labeling a graph we mean an injective map defined from the set of vertices to the set of natural numbers, the same is extended to the set of edges. The labeling from the set of vertices and edges to the set of natural numbers, such that the sum of labels of vertex and the edges incident to that vertex is a constant. A Vertex bimagic total labeling of a graph G is a function $f: V \cup E \rightarrow \{1,2,\ldots,p+q\}$ such that for every vertex $v, f(u) + p_v \in (N(u)), f(uv) = K_1 + K_2$, A graph with vertex magic total labeling with two constants K_1 or K_2 is called a vertex bimagic total labeling and denoted by VBTL[7][8].

RESULTS

Complementary perfect triple connected domination number for some standard graphs
Here we describe complementary perfect triple connected domination number for some standard graphs are given below:
(1) For any cycle of order $p \geq 5$, $\gamma_{cptc}(C_p) = p - 2$.
(2) For any complete bipartite graph of order $p \geq 5$,

$$\gamma_{cptc}(Km,n) = \begin{cases} 3, \text{if p is odd} \\ 4, \text{if p is even} \end{cases}$$

where $m, n \geq 2$ and $m + n = p$.
(3) For any book graph of order $p \geq 6$, $\gamma_{cptc}(B_p) = 4$
(4) For any friendship graph of order ≥ 5, $\gamma_{cptc}(F_p) = 3$.
(5) For any wagner graph $p \geq 8$, $\gamma_{cptc}(W_p) = 4$
3. Magic labelings for CPTC domination number for some standard graphs

Wagner graph
Let us label the function as follows, $f: V(G) \cup E(G) \rightarrow \{1,2,3,\ldots,p+q\}$. $f(V_1) = 20$,
$f(v_1) = 20, f(v_2) = 19, f(v_3) = 18, f(v_4) = 17, f(v_5) = 16, f(v_6) = 15, f(v_7) = 14,$
$f(v_8) = 13$.
$f(e_1) = 1, f(e_2) = 2, f(e_3) = 3, f(e_4) = 4, f(e_5) = 5, f(e_6) = 6, f(e_7) = 7,$
$f(e_8) = 8, f(e_9) = 9, f(e_{10}) = 10, f(e_{11}) = 11, f(e_{12}) = 12$.

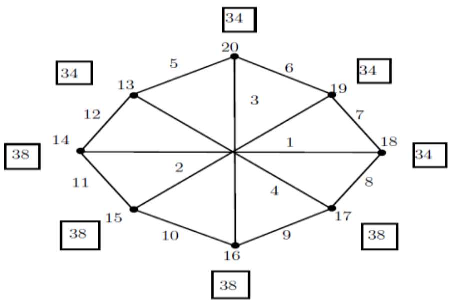

Fig 3.1 Wagner graph $\gamma_{cptc}(W_8) = 4$

Here $w_f(v_i)$ represent the vertex weight, (where $i = 1$ to 20) in Fig 3.1. In this graph the integers from 1 to 20 are used such that, the vertex labeled 13,18,19,20 has the magic constant 34 and the vertex labeled 14,15,16,17 has the magic constant 38. This graph has vertex bimagic total labeling with constant $K_1 = 34$ and $K_2 = 38$ .Hence the graph G has Vertex bimagic total labeling with $\gamma_{cptc}(W_8) = 4$.

Friendship graph
Let us label the function as follows, $f: V(G) \cup E(G) \rightarrow \{1,2,3,\ldots,p+q\}$.
$f(v_1) = 1, f(v_2) = 11, f(v_3) = 12, f(v_4) = 13, f(v_5) = 14, f(v_6) = 15, f(v_7) = 16.$
$f(e_1) = 2, f(e_2) = 3, f(e_3) = 4, f(e_4) = 5, f(e_5) = 6, f(e_6) = 7, f(e_7) = 8,$
$f(e_8) = 9, f(e_9) = 10$.

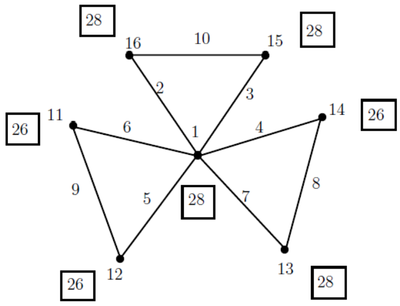

Fig 3.2 Friendship graph $\gamma_{cptc}(F_7) = 3$

Here $w_f(v_i)$ represent the vertex weight, (where i= 1 to 16) in Fig 3.2. In this graph the integers from 1 to 16 are used such that, the vertex labeled 1,13,15,16 has the magic constant 28 and the vertex labelled 11,12,14 has the magic constant 26. This graph has vertex bimagic total labeling with constant $K_1 = 26$ and $K_2 = 28$. Hence the graph G has Vertex bimagic total labeling with $\gamma_{cptc}(F_7) = 3$.

Complete bipartite graph

Let us label the function as follows, $f: V(G) \cup E(G) \to \{1,2,3,\dots,p+q\}$.
$f(v_1) = 6, f(v_2) = 10, f(v_3) = 4, f(v_4) = 7, f(v_5) = 11$.
$f(e_1) = 8, f(e_2) = 2, f(e_3) = 3, f(e_4) = 5, f(e_5) = 1, f(e_6) = 9$.

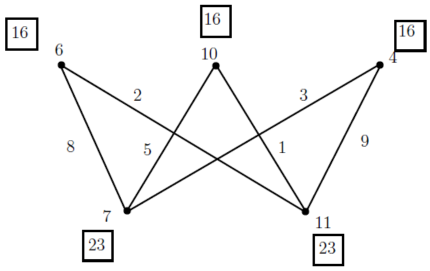

Fig 3.3 Complete bipartite graph $\gamma_{cptc}(K_{3,2}) = 3$

Here $w_f(v_i)$ represent the vertex weight, (where i= 1 to 11) in Fig 3.3. In this graph the integers from 1 to 11 are used such that, the vertex labeled 6,10,4 has the magic constant is 16 and the vertex labeled 7,11 has the magic constant 23. This graph has vertex bimagic total labeling with constant $K_1 = 16$ and $K_2 = 23$. Hence the graph G has Vertex bimagic total labeling with $\gamma_{cptc}(K_{3,2}) = 3$.

Book graph

Let us label the function as follows, $f: V(G) \cup E(G) \to \{1,2,3,\dots,p+q\}$.
$f(v_1) = 10, f(v_2) = 12, f(v_3) = 9, f(v_4) = 8, f(v_5) = 7, f(v_6) = 13$.
$f(e_1) = 1, f(e_2) = 2, f(e_3) = 3, f(e_4) = 4, f(e_5) = 5, f(e_6) = 6, f(e_7) = 11$.

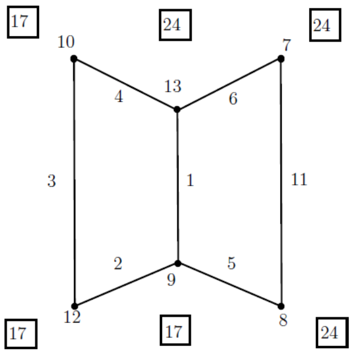

Fig 3.4 Book graph $\gamma_{cptc}(B_6) = 4$

Here $w_f(v_i)$ represent the vertex weight, (where i= 1 to 13) in Fig 3.4. In this graph the integers from 1 to 13 are used such that, the vertex labeled 9,10,12 has the magic constant is 17 and the vertex labeled 7,8,13 has the magic constant 24. This graph has vertex bimagic total labeling with constant k1= 17 and k2= 24 .Hence the graph G has Vertex bimagic total labeling with $\gamma_{cptc}(B_6) = 4$.

Cycle graph

Let us label the function as follows, $f: V(G) \cup E(\mathrm{G}) \rightarrow \{1,2,3,\ldots,p+q\}$.

$f(v_1) = 8, f(v_2) = 7, f(v_3) = 4, f(v_4) = 10, f(v_5) = 9$

$f(e_1) = 2, f(e_2) = 3, f(e_3) = 5, f(e_4) = 1, f(e_5) = 6$

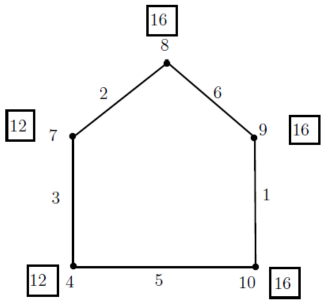

Fig 3.5 Cycle graph $\gamma_{cptc}(C_5) = 3$

Here $w_f(v_i)$represent the vertex weight, (where i= 1 to 10) in Fig 3.5. In this graph the integers from 1 to 10 are used such that, the vertex labeled 8,9,10 has the magic constant 16 and the vertex labeled 7,4 has the magic constant 12. This graph has vertex bimagic total labeling with constant k1= 16 and k2= 12 .Hence the graph G has Vertex bimagic total labeling with $\gamma_{cptc}(C_5) = 3$.

CONCLUSION

VBTL on Complementary perfect triple connected domination number for some standard graphs have been discussed in this paper. Further we studied VBTL for many other graphs which exists Complementary perfect triple connected domination number.

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